Reg No.:_____   Name:_____

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY
THIRD SEMESTER B.TECH DEGREE EXAMINATION(R&S), DECEMBER2019
Course Code: MA201
Course Name: LINEAR ALGEBRA AND COMPLEX ANALYSIS

Max. Marks: 100   Duration: 3 Hours

PART A
Answer any two full questions, each carries 15 marks

Marks

1

a) Check whether the function $f(z) = \begin{cases} Re\left(\frac{z^2}{|z|}\right), & z \neq 0 \\ 0, & z = 0 \end{cases}$ is continuous at $z = 0$. (7)

b) Show that if $f(z) = u(x,y) + iv(x,y)$ is analytic, then $u(x,y)$ and $v(x,y)$ satisfy Cauchy- Riemann equations. (8)

2 a) Determine the region in the w −plane into which the triangular region bounded by $x = 1$, $y = 1$ and $x + y = 1$ is mapped by $w = z^2$. (7)

b) Find the linear fractional transformation that maps $(-2, 0, 2)$ onto $\left(\infty, \frac{1}{4}, \frac{3}{8}\right)$. Under this transformation what is the image of the x −axis. (8)

3 a) Find the real part of an analytic function whose imaginary part is $v = e^{-x}(x\cos y + y\sin y)$. Also find the corresponding analytic function. (7)

b) Prove that $w = \frac{z}{1-z}$ maps the upper half plane $y > 0$ into the upper half plane of w −plane. What is the image of $|z| = 1$ under this mapping? (8)

PART B
Answer any two full questions, each carries 15 marks

4 a) Use Cauchy's Integral formula to evaluate $\oint_C \frac{z^2+1}{z^2-1}\,dz$ counter clock wise around (7)
$(i)|z - 1| = 1 \qquad (ii)|z + 1| = 1$

b) Find the Laurent's series of $\frac{1}{(z-1)(z-2)}$ in (8)
$(i)\ 1 < |z| < 2 \quad (ii)\ |z| > 2 \quad (ii)\ 0 < |z - 1| < 1$

5 a) Use Cauchy's Residue theorem to evaluate $\oint_C \left(\frac{Ze^{\pi Z}}{Z^4-16}\right)dz$, where C is the ellipse $9x^2 + y^2 = 9$. (7)

b) Evaluate $\int_0^{2\pi} \frac{d\theta}{\sqrt{2}-\cos\theta}$ using contour integration. (8)

6 a) Evaluate $\int (Re\ z)\,dz$ along the real axis from 0 to 1 and then along a straight line parallel to imaginary axis from 1 to $1 + 2i$. (7)

b)   Evaluate $\int_{-\infty}^{\infty} \frac{1}{(x^2+1)^2} dx$ using contour integration.   (8)

PART C

Answer any two full questions, each carries 20 marks

7   a)   Solve the system of equations using Gauss Elimination method:

$$y + z - 2w = 0, \qquad 2x - 3y - 3z + 6w = 2, \qquad 4x + y + z - 2w = 4$$
    (8)

b)   If the matrix $\begin{bmatrix} 1 & -2 & 3 & 1 \\ 2 & 1 & -1 & 2 \\ 6 & -2 & a & b \end{bmatrix}$ is of rank **2**, find the values of a, b.   (6)

c)   Check whether the vectors $[1,\ 2,\ 1]$, $[2,\ 1,\ 4]$, $[4,\ 5,\ 6]$, $[1,\ 8,-3]$ are linearly dependent in R^3.   (6)

8   a)   Diagonalise the symmetric matrix $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ (8)

b)   If one eigen values of the matrix $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ is **5,** find the other eigen values without finding the characteristic equation. What are the eigen values of A^2 and A^{-1}.   (6)

c)   Reduce the quadratic form $q = 3x^2 + 5y^2 + 3z^2 - 2yz + 2zx - 2xy$ to the canonical form. Examine the definiteness.   (6)

9   a)   Find a matrix B which transform $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$ in to the diagonal form.   (10)

b)   Find a basis and dimension for the row space, column space and null space for the matrix $A = \begin{bmatrix} 1 & 2 & 0 & 2 & 5 \\ -2 & -5 & 1 & -1 & -8 \\ 0 & -3 & 3 & 4 & 1 \\ 3 & 6 & 0 & -7 & 2 \end{bmatrix}$ (10)

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